MOCK FINAL EXAM

PEYAM RYAN TABRIZIAN

Name: ________________________

Instructions: You have 3 hours to take this exam. It is meant as an opportunity for you to take a real-time practice final and to see which topics you should focus on before the actual final! Even though it counts for 0% of your grade, I will grade it and comment on it overnight, and you can pick up the graded exam tomorrow at noon in my office (830 Evans)

Note: Questions $14 - 17$ are a bit more challenging (although not impossible) than the rest! They are meant to be an extra challenge for people who finish early (and hence they are only worth 5 points each)

Note: Please check one of the following boxes:

- ☐ I will pick up my exam tomorrow between noon and 5 pm, and I want comments on my exam (Peyam Tabrizian approves of this choice :))
- ☐ I will pick up my exam tomorrow between noon and 5 pm, but I don't want comments on my exam (I only want to know my score)
- ☐ I will not pick up my exam tomorrow, just grade it and enter my score on bspace!

1		15
2		10
3		20
4		10
5		15
6		15
7		10
8		10
9		15
10		10
11		20
12		15
13		15
14		5
15		5
16		5
17		5
Total		200

Date: Monday, May 9th, 2011.

1. *(15 points, 3 points each)* Evaluate the following integrals:

(a) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(b) $\int_{-1}^1 \frac{\sin(x^3)(x^2+7x^6+1)}{\cos(x)+2} dx$

(c) $\int_{-2}^0 \sqrt{4-x^2} dx$

(d) $\int \frac{\cos(x)}{\sin^2(x)} dx$

(e) $\int_1^2 \frac{\ln(x)}{x} dx$

2. *(10 points)*

(a) *(8 points)* Show that the function $f(x) = \cos(x) - x$ has at least one zero.

(b) *(2 points)* Using part (a), show that the function $g(x) = \sin(x) - \frac{x^2}{2}$ has at least one critical point.

3. *(20 points)* Sketch a graph of the function $f(x) = x\ln(x) - x$. Your work should include:
   - Domain
   - Intercepts
   - Symmetry
   - Asymptotes (no Slant asymptotes, though)
   - Intervals of increase/decrease/local max/min
   - Concavity and inflection points

(This page is left blank in case you need more space to do question 3.)

4. *(10 points)* Using the definition of the integral, evaluate $\int_0^2 \left(x^3 + x\right) dx$. You may use the facts that:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

5. *(15 points, 5 points each)* Evaluate the following limits:

(a) $\lim_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x}\right)$

(b) $\lim_{x\to -\infty} \frac{\sqrt{x^2+1}}{x}$

(c) $\lim_{x\to \infty} \left(1+\frac{2}{x}\right)^x$

6. *(15 points)* Find the area between the curves $4x + y^2 = 12$ and $x = y$

7. *(10 points)* Suppose f is an odd function and is differentiable everywhere. Prove that, for every positive number b, there exists a number c in $(-b, b)$ such that $f'(c) = \frac{f(b)}{b}$

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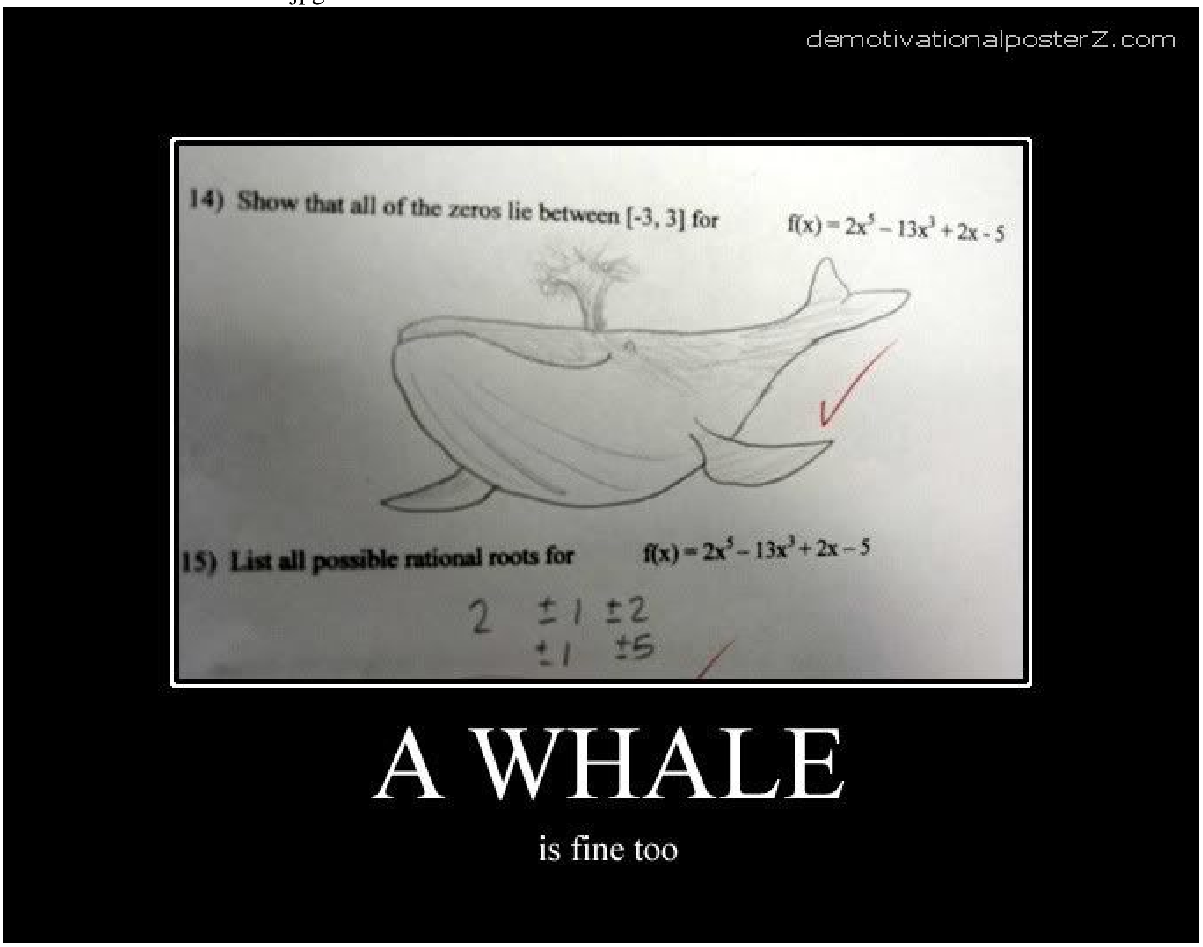

8. *(10 points)* Find the volume of the solid obtained by rotating the region bounded by $y = x$, $y = \sqrt{x}$ about $y = 1$

9. *(15 points)* A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline, and the angular velocity of the light is 8π radians per minute. How fast is the beam of light moving along the shoreline when it is 1 km away from P?

10. *(10 points, 5 points each)* Find the derivatives of the following functions:

(a) $f(x) = \sin^{-1}(x)\sqrt{1-x^2}$

(b) $f(x) = x^{\ln(x)}$

11. *(20 points)* Find the volume of the donut obtained by rotating the disk of center $(3,0)$ and radius 2 about the $y-$axis.

12. *(15 points)* Show that the equation of the tangent line to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x_0, y_0) is:

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

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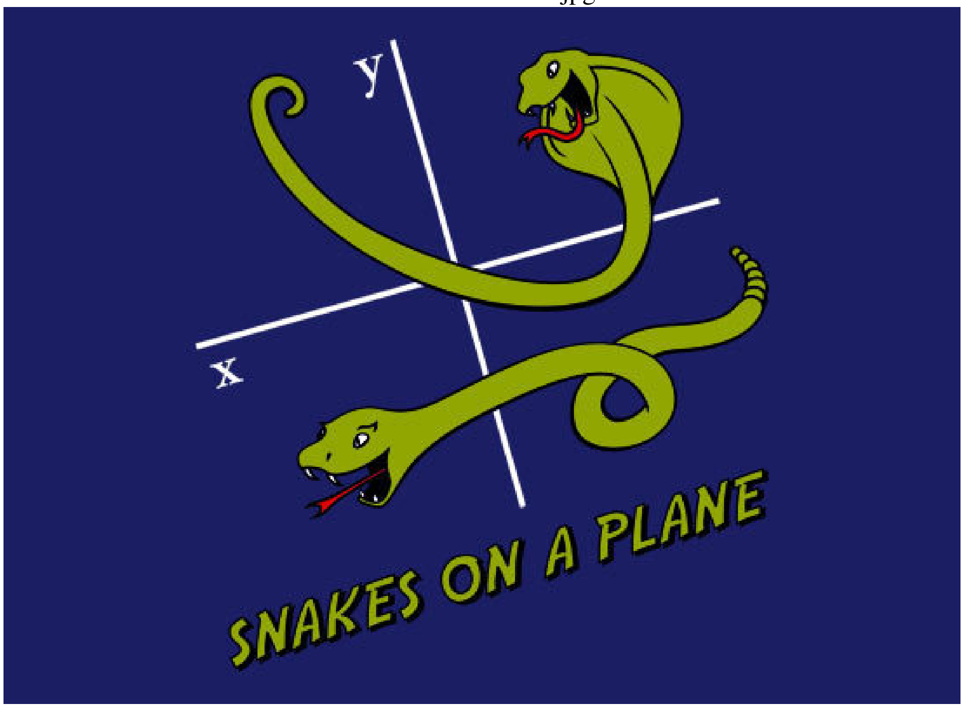

13. *(15 points)* Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

Note: I would like to remind you that questions $14 - 17$ are more challenging than the rest, but you can give them a try if you want to, they are not impossible to do!

14. *(5 points)* Solve the differential equation $T' = T - 5$.

    Hint: Let $y = T - 5$. What differential equation does y solve?

15. *(5 points)* If f is continuous on $[0,1]$, show that $\int_0^1 f(x)dx$ is finite.

16. *(5 points)*
   (a) Use l'Hopital's rule to show:

$$\lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x)$$

   (b) Use (a) to answer the following question: If $f(x) = x^2 \sin(\frac{1}{x})$ with $f(0) = 0$, does $f''(0)$ exist?

17. *(5 points)* If f is differentiable (except possibly at 0) and $\lim_{x\to\infty} f(x) = 0$, is it true that $\lim_{x\to\infty} f'(x) = 0$? Prove it or give an explicit counterexample!

You're done!!!

Any comments about this exam? (too long? too hard?)

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